B.A/B.Sc. 5th Semester (Honours) Examination, 2019 (CBCS)

Subject : Mathematics

Paper : BMH5DSE21

(Probability and Statistics)

Time: 3 Hours **Full Marks: 60**

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meaning.

1. Answer *any ten* questions: 2×10=20

(a) If A and B be two events such that $A \subseteq B$, then prove that $P(A) \leq P(B)$.

(b) The radius X of a circle has uniform distribution in $(1,2)$. Find the variance of the area of the circle.

(c) Show that the standard deviation is independent of any change of origin but dependent on change of scale.

(d) If the regression equation of Y on X be $y = 0{\cdot}57x + 6{\cdot}93$ and X on Y be $x = 1{\cdot}12y - 2{\cdot}5$, find the correlation coefficient between X and Y.

(e) State Bernoulli's limit theorem.

(f) Write down the maximum likelihood function for the normal (m, σ) population.

(g) If $\bar{x}$ is an unbiased estimator of population mean μ, examine whether $\bar{x}^2$ is an unbiased estimator of μ^2 or not.

(h) What do you mean by confidence interval of a population parameter θ?

(i) Distinguish between type-I and type-II errors in testing of hypothesis.

(j) A random variable X assumes the values –1, 0, 1 with probabilities $\frac{1}{3}, \frac{1}{2}, \frac{1}{6}$ respectively. Find the cumulative distribution function of X.

(k) If the probability density function of a random variable is given by

$$f(x) = Ke^{-(x^2+2x+3)}, \quad -\infty < x < \infty.$$

Find the value of the constant K.

(l) The random variable X has mean 2 and variance 4. Find the variance of $Y = \frac{1}{2}X$.

(m) Show that the sample mean is a consistent estimate of the population mean.

(n) Give the definition of convergence in Probability.

(o) Find the moment generating function of a Binomial distribution.

2. Answer *any four* questions from the following: 5×4=20

(a) Define distribution function of a random variable X and prove that it is a monotonic non-decreasing function of real variable x. 1+4=5

(b) Define characteristic function of a random variable X.

Find the characteristic function of the distribution whose probability density function is given by $f(x) = \begin{cases} 1-|x|, & \text{if } |x|<1 \\ 0, & \text{elsewhere} \end{cases}$. 1+4=5

(c) Prove that first absolute moment of a continuous random variable about any point is minimum when taken about median. 5

(d) The joint probability density function of two random variables X and Y is $K(1-x-y)$ inside the triangle formed by the coordinate axes and the line $x+y=1$ and zero elsewhere. Find the value of K and $P\left(X<\frac{1}{2}, Y>\frac{1}{4}\right)$. Also, find the marginal density function of X. 2+2+1=5

(e) Find the maximum likelihood estimate of θ in $f(x) = ke^{-x/\theta}, x>0, k>0$ on the basis of a random sample of size n. 5

(f) What is sampling distribution of a statistic? Show that the function

$$t = \frac{\sqrt{u}(\bar{X}-m)}{s}$$

is t-distribution with $\nu = n-1$ degrees of freedom. 1+4=5

3. Answer *any two* questions from the following: 10×2=20

(a) (i) The probability density function of a random variable X is

$$f(x) = \frac{1}{4}|x|, \quad -2<x<2$$
$$= 0, \quad \text{elsewhere}$$

Find the distribution function of X.

(ii) A coin is tossed repeatedly and the probability that a head appears at any toss is p where $0<p<1$. Find the expected length of the initial run of heads.

(iii) The probability density function of a random variable X is given by

$$f(x) = 2xe^{-x^2}, \quad x>0$$
$$= 0, \quad \text{elsewhere}$$

find the probability density function of X^2. 3+3+4=10

(b) (i) For any pair of correlated random variables X and Y, we make a linear transformation $(X,Y) \to (U,V)$, given by a rotation of the axes through a constant angle α, i.e.

$U = X\cos\alpha + Y\sin\alpha$

$V = -X\sin\alpha + Y\cos\alpha$

Then show that U and V will be uncorrelated if $\tan 2\alpha = \frac{2\rho\sigma_x\sigma_y}{\sigma_x^2-\sigma_y^2}$.

where $\rho = \rho(X,Y)$.

(ii) Show that if X is a binomial (n, p) variate then $\frac{X-np}{\sqrt{npq}}$ tends to a standard normal variate as $n \to \infty$.

5+5=10

(c) (i) State and prove Tchebycheff's inequality.

(ii) Show by Tchebycheff's inequality that in 2,000 throws with a coin the probability that the number of heads lies between 900 and 1,100 is at least 19/20.

(iii) What is the difference between (α) X_n converges to X in probability and (β) X_n converges to X in distribution?

4+3+3=10

(d) (i) If the sample observations are 1, 3, 4, 5, 8, 9 draw from an infinite population with variance σ^2, find an unbiased estimator of σ^2.

(ii) A sample be drawn from normal population with variance 10·24. The observations are 20, 22, 18, 27, 24, 30, 21, 19 and 26. Obtain a 95% confidence interval for population mean. It is given that $\frac{1}{\sqrt{2\pi}} \int_{1 \cdot 96}^{\infty} e^{-\frac{1}{2}x^2} dx = 0 \cdot 025$.

(iii) A fertiliser mixing machine is set to give 12 kg of nitrate for every quintal bag of fertiliser. Ten 100-kg bags are examined. The percentages of nitrate are : 11, 14, 13, 12, 13, 12, 13, 14, 11, 12. Give reason that the machine is defective.

(Value of t for 9 degrees of freedom is 2·262 at 5% level of significance.) 3+5+2=10
